TI-68k vs HP49G+ Benchmark

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Machine configuration

- Timing for the 68k was done on my Voyage 200:

  AMS 2.09

  168 KB free RAM, empty history area, no TSRs or kernels installed

  Mode settings: DisplayDigits → Float12, Angle → Radian, ExponentialFormat → Normal, ComplexFormat → Rectangular, PrettyPrint → On, Exact/Approx → Auto.

- HP49G+:

  ROM revision 1.23, CAS version 4.20031005

  233KB + 127KB free RAM

  Flag settings: {#8205010FF0h, #0h, #880404402028000h, #0h}

Timings

- Time is given in seconds, unless otherwise noted
- A blank square means that the timing has not yet been done for that example
- Like other computer algebra benchmarks, the CAS timings include evaluation time but not display time
- The graphics performance benchmark includes computation time as well as rendering time
- The display-routine performance benchmark includes conversion/printing time but not evaluation time

Other notes

- Free add-on programs are included in this benchmark
- Performance comparisons are done only when both machines have that particular functionality
- For the most part, this benchmark is currently focused on math problems, not applications such as engineering

- Inputs are normally given in TI syntax
- For floating point arithmetic, approximate values of the arguments shown are used
- Most of the CAS timing examples were chosen without evaluating them on either calculator
- A blank square in the functionality comparison means that the calculator does not have the functionality

To do

- Include ASI, numerical inverse program, and HP49G+ programs, and add references to add-ons used
- This might be a good chance to make a set of regression tests (the same as or similar to the examples below)
- A few examples (e.g. for polynomial factorization) may have been from Bernard Parisse; add references if needed
- Consider including non-doable ("give-up") examples in CAS performance table
- Add to functionality table:

  Special functions (Airy, Euler numbers, Fibonacci, Fresnel, erf, exponential integrals, Hurwitz zeta, $\frac{\partial \zeta(s, a)}{\partial s}$, polylogs, hypergeometrics, orthogonal polynomials, cyclotomic, number theoretical)
- Things to try:

  Poles, essential singularities, branch cuts, and discontinuous functions for symbolic and numeric definite integration. Non-differentiable functions and discontinuous functions for optimization. Non-squarefree polynomials and large coefficients for factorization. Finite (abs, sign, min, max, when) and infinite (floor, ceiling, round, fpart, mod, intdiv) piecewise functions for integration and equation solving. Integrands of type $R\left(x, \sqrt{x^2 + b \cdot x + c}\right)$.
- Include plot settings, such as window settings (typically standard V200 settings)
- Functionality comparison:

  Systems of nonlinear polynomial equations (check how the numerical solvers handle discontinuities)

  Laurent series, asymptotic series, limits, hypergeometric summation

  Definite integration (check singularity detection, e.g. $\int(1/(\sin(x)+2),x,0,2\pi)$;compare nInt(1/(x+1),x,-3,3))

  Graphics (e.g. 68k ComplexMap vs 49G GridMap, vector plot, vector field plot, 3D parametric curves and surfaces)
- Add some semblance of a summary

Disclaimer

- It is significantly harder to develop a fair benchmark for symbolics than for numerics. The examples give only a rough idea of the strengths and weaknesses. Also, I do not know many of the methods used internally by the CAS's.

Performance

Problem	TI	HP	Comments
91356200129 + 7868039544	0.001	□	Integer arithmetic – addition
356530692 * 200980515627	0.002	□	Integer arithmetic – multiplication
7^{477}	0.17	□	Integer arithmetic – exponentiation
$\sqrt{3} + \sqrt{5}$	0.003	□	Real floating point arithmetic – addition
$\sqrt{3} * \sqrt{5}$	0.004	□	Real floating point arithmetic – multiplication
$\sqrt{3} / \sqrt{5}$	0.005	□	Real floating point arithmetic – division
$\left(\sqrt{3}\right)^{\sqrt{5}}$	0.024	□	Real floating point arithmetic – exponentiation
$\sqrt{13}$	0.009	□	Real floating point arithmetic – square root
$\cos(\pi + e)$	0.015	□	Real floating point arithmetic – cosine
□	□	□	Real floating point arithmetic – arctangent
□	□	□	Complex floating point arithmetic – addition
□	□	□	Complex floating point arithmetic – multiplication
□	□	□	Complex floating point arithmetic – division
□	□	□	Complex floating point arithmetic – exponentiation
□	□	□	Complex floating point arithmetic – square root
□	□	□	Complex floating point arithmetic – absolute value
□	□	□	Complex floating point arithmetic – natural logarithm
$\gcd\left(29^{400}, 31^{300}\right)$	11.42	7.23	Integer gcd
$180!$	0.29	0.51	Factorial
$\text{isPrime}\left(2^{127} - 1\right)$	34.95	32.42	Primality testing
factor (211!)	5.21	7.10	Integer factorization
factor (4498414682539051)	5.74	3.50	Integer factorization with moderately large factors
factor (num1)	72.16	No	Integer factorization with large factors
$\text{expand}\left((x + y)^{99}\right)$	6.47	225.3	Polynomial expansion
$\text{expand}\left(1 / \left(x^6 + 1\right), x\right)$	0.88	4.09	Partial fraction expansion
expand (1 / poly5)	12.32	□	Partial fraction expansion
SqrFree (poly6, x)	2.05	□	Squarefree factorization
$\text{factor}\left(x^{100} - 1\right)$	1.09	□	Univariate factorization
$\text{factor}\left(x^{100} - 1, x\right)$	7.26	Hang	Univariate factorization
factor (poly1)	□	□	Univariate factorization
□	□	□	Univariate factorization with parameters
factor (poly2)	□	□	Multivariate factorization
factor (ratfunc1)	□	□	Multivariate factorization
solve (3 x + 9 = 0, x)	0.06	□	Univariate polynomial solving

solve $\left(x^6 - x^4 - 4x^2 + 4 = 0, x\right)$	1.37	□	Univariate polynomial solving – real solutions
cSolve $\left(x^6 - x^4 - 4x^2 + 4 = 0, x\right)$	1.63	□	Univariate polynomial solving – complex solutions
cZeros $\left(x^4 + x^3 + x^2 + x + 1, x\right)$	9.05	□	Univariate polynomial solving – complex solutions
cZeros $\left(x^6 - 1, x\right)$	0.18	2.17	Univariate polynomial solving – complex solutions
cZeros ((x^5 – x)^33, x)	□	□	Univariate polynomial solving – complex solutions
solve $((x-7)\cdot(x-5)\cdot(x-3)=0, x) \mid x \le 6$	0.13	□	Univariate polynomial solving – constrained
cSolve (x^5 + x + 1 = 0 and abs (x) = 1, x)	□	□	Univariate polynomial solving – constrained
cSolve ($x\_^5 = x\_$ and imag (x_) ≠ 0, x_)	1.32	□	Univariate polynomial solving – constrained
□	□	□	Univariate polynomial solving – parameters
solve $\left(x^2 + y^2 = 1, \{x, y\}\right)$	0.42	□	Multivariate polynomial solving – real solutions
zeros $\left(\left\{x^3 + 3\cdot x\cdot y + y^3, x + y^3\right\}, \{x, y\}\right)$	7.19	□	Multivariate polynomial solving – real solutions
cSolve $\left(x^2 + y^2 = 1, \{x, y\}\right)$	0.25	□	Multivariate polynomial solving – complex solutions
cZeros $\left(\left\{x^3 + 3\cdot x\cdot y + y^3, x + y^3\right\}, \{x, y\}\right)$	9.05	□	Multivariate polynomial solving – complex solutions
solve (polysys, {x, y})	□	□	Polynomial system solving – real solutions
cSolve (polysys, {x, y})	□	□	Polynomial system solving – complex solutions
□	□	□	Univariate polynomial solving (numeric)
PolyGCD (poly7, poly8)	0.28	0.47	Univariate polynomial gcd
PolyGCD (poly10, x^5 – 11 x + 9)	□	□	Univariate polynomial gcd (partially factored input)
□	□	□	Multivariate polynomial gcd
□	□	□	Univariate polynomial reduction modulo a prime
□	□	□	Polynomial reduction modulo a polynomial
□	□	□	Univariate polynomial quotient and remainder
$d\left(x^{71}, x, 60\right)$	0.30	□	Differentiation – polynomials
$d\left(5\cdot x^{11} + 3\cdot x^7 - 7\cdot x^3 + 21\cdot x^2 - 1, x, 5\right)$	0.12	□	Differentiation – polynomials
□	□	□	Differentiation – rational functions
$d\left(\sqrt{x}, x, 5\right)$	0.05	2.05	Differentiation – algebraic functions (ℜ3)
$d\left(e^{a*x}, x, 100\right)$	0.82	□	Differentiation – transcendental functions
d(cos (ln (x)), x, 50)	3.32	□	Differentiation – compositions of functions
$d\left(x^{\sin(x)}, x, 3\right)$	4.63	□	Differentiation – compositions of functions
d(f (x) / g (x), x, 4)	11.47	□	Differentiation – undefined functions
limit $\left(\sin\left(\sqrt{x}\right), x, 0\right)$	0.01	0.25	Limit
limit $\left((1 + 1/x)^x, x, \infty\right)$	0.12	4.93	Limit
limit $\left(x^{12}\cdot e^{-x}, x, \infty\right)$	0.25	5.35	Limit
limit $\left(\left(x^4 - 6x^3 + x^2 + 3\right)/(x-1), x, 1\right)$	0.16	4.15	Limit – pole
limit (sin (1 / x), x, 0)	0.04	2.31	Limit – essential singularity
limit $\left(\ln(x)/\sqrt{x}, x, \infty\right)$	0.08	6.36	Limit
limit $\left(\sqrt{x+a} - \sqrt{x}, x, \infty\right)$	0.84	22.44	Limit
limit $\left(x^{22}/77^x, x, \infty\right)$	0.72	11.01	Limit

limit $\left((3^x + 5^x)^{1/x}, x, \infty\right)$	0.37	59.51	Limit
taylor $\left(x^3 + 2x, x, 3, 2\right)$	0.07	□	Taylor series – polynomials (about 2 to order 3)
taylor (tan (x), x, 10)	13.00	□	Taylor series (about zero to order 10)
taylor (ln (x + 1), x, 20)	0.63	□	Taylor series
taylor (sin (cos (x)), x, 8, 2)	12.47	□	Taylor series
taylor $\left(1 \Big/ \sqrt{1 - v^2/c^2}, v, 8\right)$	9.98	□	Taylor series
□	□	□	Taylor series
Series $\left(\frac{e^x}{x^2}, x, 0, 4\right)$	0.49	1.94	Laurent series (about zero to order 4)
Series $\left(\frac{1}{x^2+a^2}, x, \infty, 0\right)$	1.96	1.01	Asymptotic series
Series $\left(e^{1/x}, x, \infty, 5\right)$	0.20	1.20	Asymptotic series
taylor $\left(\int_0^{\log(z)} e^{\sin(x)}\, dx, z, 4, 1\right)$	□	□	Series of definite integrals
$\int (x + a)^{100}\, dx$	0.027	236.9	Indefinite integration – factored polynomials
$\int \left((x - 1)^{26} + \left(\frac{x+\sqrt{5}}{2}\right)^{77}\right) dx$	0.027	115.8	Indefinite integration – partially factored polynomials
$\int \text{poly9}\, dx$	3.05	□	Indefinite integration – high degree polynomials
$\int (\sin (x) \cdot \cos^n (x))\, dx$	0.38	No	Indefinite integration – $f(x) \cdot f'(x)$
$\int \left(\cos (\ln (x)) \Big/ \left(x \cdot \sqrt{\sin (\ln (x))}\right)\right) dx$	0.14	2.80	Indefinite integration – $f'(x) / f(x)$
$\int \left(1 \Big/ \left(x^8 + 1\right)\right) dx$	19.26	No	Indefinite integration – rational functions
$\int \left(\frac{x^2+1}{x^4+x^2+1}\right) dx$	1.46	1.78	Indefinite integration – rational functions
Horowitz $\left(\text{poly3}, (x^7 - x + 1)^2, x\right)$	□	2.09	Indefinite integration – rational functions
$\int \sqrt{x + \sqrt{x}}\, dx$	□	Hang	Indefinite integration – algebraic functions
$\int \left(x^2 \cdot \left(a \cdot x^3 + b \cdot x^2\right)^{1/3}\right) dx$	9.05	No	Indefinite integration – algebraic functions
$\int \left(x^5 \Big/ \left(4x^2 + 9\right)^{1/3}\right) dx$	0.87	23.49	Indefinite integration – algebraic functions
$\int \left(x \Big/ \sqrt{a \cdot x^2 + b \cdot x + c}\right) dx$	1.04	15.53	Indefinite integration – algebraic functions
$\int \left(1 \Big/ \left(\sqrt{x} + x^{1/3}\right)\right) dx$	0.53	5.99	Indefinite integration – algebraic functions
$\int \left(x \Big/ \sqrt{1 - x^4}\right) dx$	0.23	No	Indefinite integration – algebraic functions
$\int \left(1 - \sqrt{x}\right)^{p-1} dx$	1.02	No	Indefinite integration – algebraic functions (G & R-based)
$\int \left(\frac{1}{\sqrt{x^2+1}} + \frac{100}{\sqrt{x^2+10000}}\right) dx$	0.23	7.09	Indefinite integration – algebraic functions (R4)
$\int (\sin (a \cdot x + b) \cdot \sin (x)^5)\, dx$	3.16	33.83	Indefinite integration – transcendental functions

$\int \left(\sin(x)^{12} \cdot \cos(x)^{29}\right) dx$	11.2	7.15	Indefinite integration – transcendental functions
$\int \left(e^x \cdot \sin^2(x)\right) dx$	0.13	2.34	Indefinite integration – transcendental functions
$\int \left(\frac{e^x}{e^{3x}-1}\right) dx$	1.31	3.69	Indefinite integration – transcendental functions
$\int \left(\ln(x)^4 / x^m\right) dx$	1.03	No	Indefinite integration – transcendental functions
$\int \sin(\log(x))\, dx$	□	□	Indefinite integration – transcendental functions
$\int (x \cdot a^x \cdot \cos(x))\, dx$	0.94	8.61	Indefinite integration – transcendental functions
$\int \left(x \cdot \sinh^{-1}(a/x)\right) dx$	0.62	No	Indefinite integration – transcendental functions
$\int \left(\tan^{-1}\left(\frac{x-5}{x-3}\right)\right) dx$	1.10	3.56	Indefinite integration – transcendental functions (ℜ1)
$\int \left(\frac{\sqrt{5}\, x^2}{3} - 1\right) \sqrt{-3x^2 + 4x + 2\sqrt{3}}\ dx$	7.89	12.44	Indefinite integration – mixed functions
$\int \sqrt{\tan(x)}\ dx$	Hang	5.40	Indefinite integration – mixed functions
$\int \left(f(x)^{g(x)} \cdot \left(\frac{f'(x) \cdot g(x)}{f(x)} + \ln(f(x)) \cdot g'(x)\right)\right) dx$	1.00	□	Indefinite integration – undefined functions
□	□	□	Definite integration – polynomials
$\int_{a-1}^{a+1} \frac{1}{(x-a)^{999}}\ dx$	0.29	Hang	Definite integration – rational functions
$\int_1^\infty \frac{1}{z^n}\ dz \mid n > 1$	□	□	Definite integration – rational functions
□	□	□	Definite integration – rational functions
$\int_{-r}^{r} \sqrt{x^2 - r^2}\ dx$	0.84	4.45	Definite integration – algebraic functions
$\int_0^1 \left(1 - \sqrt{x}\right)^{p-1} dx \mid p > 0$	□	□	Definite integration – algebraic functions (ℜ5)
$\int_{-\infty}^{a} x \cdot e^x\, dx$	0.32	5.35	Definite integration – transcendental functions
$\int_0^{\pi/4} (\tan^5(x))\, dx$	62.47	4.70	Definite integration – transcendental functions (ℜ5)
$\int_0^{\pi} \frac{\sin^3(x)}{p + q\cos(x)}\ dx$	3.63	17.93	Definite integration – transcendental functions (ℜ5)
$\int_0^\infty t^2 \cdot e^{-s*t}\, dt \mid s > 0$	1.47	□	Definite integration – transcendental functions
$\int_{-2}^{2} (\text{abs}(\cos(z)))\, dz$	12.43	3.60	Definite integration – piecewise functions
□	□	□	Definite integration – piecewise functions
$\int_0^\infty (f(t)/t)\, dt$	0.16	2.56	Definite integration – undefined functions
$\int_{-\infty}^{\mu} \left(x \cdot e^{(x-\mu)/\sigma} / (2\sigma)\right) dx \mid \sigma > 0$	□	□	Definite integration – other improper integrals
$\int_0^\infty \left(x \cdot e^{-x^2}\right) dx$	0.43	1.52	Definite integration – other improper integrals
$\int_0^\infty \frac{(1+x)^{p-1}}{(x+a)^{p+1}}\ dx \mid a > 0$	□	□	Definite integration – other improper integrals (ℜ5)
$\int_0^\infty \text{sech}(a \cdot x)\, dx \mid a > 0$	□	□	Definite integration – other improper integrals (ℜ5)
$\int_{-\infty}^{\infty} \frac{x^2+1}{x^4+x^2+1}\ dx$	2.94	6.91	Definite integration – contour-type improper integrals
$\text{CPVInt}\left(1/\left(5x^4 + 6x\right), x, -1, 1\right)$	5.87	3.67	Definite integration – principal value integrals

CPVInt (tan (x), x, $\pi/4$, $3\pi/4$)	4.38	3.83	Definite integration – principal value integrals
CPVInt (1 / (x · ln (x)), x, 1 / 2, 2)	5.60	3.16	Definite integration – principal value integrals
$\int_{-1}^{1}\int_{-1}^{1} 1\, d\,x\, d\,y$	0.02	0.65	Definite integration – multidimensional
$\int_{0}^{4}\int_{0}^{2*\sqrt{z}}\int_{0}^{\sqrt{4z-y^2}} 1\, d\,x\, d\,y\, d\,z$	0.76	9.25	Definite integration – multidimensional
$\int_{0}^{1}\int_{0}^{x}\left(x^2+y^2\right) d\,y\, d\,x$	0.11	1.78	Definite integration – multidimensional
$\text{nInt}\left(x^2-2x+3,\ x,\ 0,\ 1\right)$	1.20	?	Numeric integration – polynomials
$\text{nInt}\left(1\,/\left(x^4+x^2+9/10\right),\ x,\ -1,\ 1\right)$	□	□	Numeric integration – rational functions
nInt (sec (tan (x)), x, 0, 1)	□	□	Numeric integration – finite intervals
nInt (ln (x), x, −1, 1)	□	No	Numeric integration – finite intervals
nInt (cos (x), x, −10, 10)	□	13.8	Numeric integration – even integrands
nInt (when (x ≥ 0 and x < 0.3, 0, 1), x, 0, 1)	□	□	Numeric integration – piecewise integrands
nInt (min (sin (x), cos (x)), x, $-3\pi/2$, $3\pi/2$)	□	Hang	Numeric integration – piecewise integrands
$\text{nInt}\left(1\,/\,\sqrt{\lvert x\rvert},\ x,\ -1,\ 1\right)$	□	□	Numeric integration – singularity
$\text{nInt}\left(1\,/\,x^8,\ x,\ -\pi,\ \pi\right)$	□	□	Numeric integration – singularity
$\text{nInt}\left(\frac{x\sin(x)}{1-\cos(x)},\ x,\ 0,\ \pi/2\right)$	□	□	Numeric integration – singularity at endpoint (ℝ5)
$\text{nInt}\left(\sqrt{(x+1)\cdot(x-1)}\cdot\ln(x),\ x,\ 0,\ 1\right)$	□	□	Numeric integration – singularity at endpoint
$\text{nInt}\left(x^3\cdot\ln\left(\left\lvert\left(x^2-1\right)\left(x^2-2\right)\right\rvert\right),\ x,\ 0,\ 3\right)$	□	□	Numeric integration – singularity
$\text{nInt}\left(\sin(x)\,/\left(x^{100}+1\right),\ x,\ 1,\ \infty\right)$	□	□	Numeric integration – oscillatory integrands
$\text{nInt}\left(\frac{1}{10\sqrt{2\pi}}\cdot e^{-\frac{(x-20)^2}{2*10^2}},\ x,\ 104,\ 136\right)$	7.51	□	Numeric integration – Gaussians
$\text{nInt}\left(1\,/\,\sqrt{x},\ x,\ 0,\ \infty\right)$	□	□	Numeric integration – other improper integrals
$\text{nInt}\left(\ln(x)\,/\left(10\cdot x^2+1\right),\ x,\ 0,\ \infty\right)$	□	□	Numeric integration – other improper integrals
$\text{nInt}\left(\text{nInt}\left(\text{abs}\,(x-y)^{3/2},\ x,\ 0,\ 1\right),\ y,\ 0,\ 1\right)$	□	□	Numeric integration – multidimensional
Laplace ($t^7\cdot\sin(t)$, t, s)	□	□	Laplace transform
$\text{ILaplace}\left(\frac{1}{1+s^2+s^4},\ s,\ t\right)$	□	□	Inverse Laplace transform
□	□	□	Linear constant coefficient ODE
$\text{deSolve}\left(y' = x\cdot y^2,\ x,\ y\right)$	□	□	Linear first-order variable coefficient ODE
deSolve ($y' - x\cdot y = 1$, x, y)	□	□	Linear first-order variable coefficient ODE
$\text{deSolve}\left(x^2\cdot y'' + a\cdot x\cdot y' = \ln(x),\ x,\ y\right)$	□	□	Linear second-order variable coefficient ODE
deSolve ($y\cdot y' = 1$, x, y)	□	□	Nonlinear first-order ODE
$\text{deSolve}\left(x^5\cdot y' + y^2 - 4x^4\cdot y + x^8 = 0,\ x,\ y\right)$	□	□	Nonlinear first-order ODE (Riccati-type)
deSolve (ode1, x, y)	□	□	Nonlinear first-order ODE (Kamke 1.505)
$\text{deSolve}\left(y'' + y\cdot(y')^3 = 0,\ x,\ y\right)$	□	□	Nonlinear second-order ODE
$\text{deSolve}\left(8\cdot y'' + 9\cdot(y')^4 = 0,\ x,\ y\right)$	□	□	Nonlinear second-order ODE
$\text{deSolve}\left(y'' = a\cdot\sqrt{(y')^2+1},\ x,\ y\right)$	□	□	Nonlinear second-order ODE
deSolve ($y'' = e^y$, x, y)	□	□	Nonlinear second-order ODE

deSolve (y" = a · y' + y and initcond1, x, y)	□	□	Initial value problem
deSolve (y" = - y– x and boundcond1, x, y)	□	□	Boundary value problem
□	□	□	Numerical ODE solving with initial conditions
$\text{solve}\left(x^{1/3} + \sqrt{x} = 1, x\right)$	□	□	Algebraic equation solving
$\text{solve}\left(\ln\left(\sqrt{x}\right) = \sqrt{\ln(x)}, x\right)$	□	□	Transcendental equation solving
$\text{solve}(2 \cdot \sin(x) \cdot \cos(x) = 3, x)$	□	□	Transcendental equation solving – real solutions
$\text{solve}\left(e^{\sin(x)} = 2, x\right)$	□	□	Transcendental equation solving – real solutions
$\text{solve}\left(e^{\sin(x)} = 2, x = 30\right)$	□	□	Transcendental equation solving – one real solution
$\text{cSolve}(2 \cdot \sin(x) \cdot \cos(x) = 3, x)$	□	□	Transcendental equation solving – complex solutions
$\text{cSolve}\left(e^{\sin(x)} = 2, x\right)$	□	□	Transcendental equation solving – complex solutions
$\text{cSolve}\left(e^{\sin(x)} = 2, x = -3 \cdot i\right)$	□	□	Transcendental equation solving – one complex solution
$\text{solve}\left(\cosh^{-1}\left(z^2 - a \cdot z\right) = a, z\right)$	□	□	Transcendental equation solving – parameters
$\text{Resultnt}\left(x^3 - x \cdot y + z, 3x^2 - y, x\right)$	0.81	0.54	Univariate resultant
$\text{fMin}\left(x^2 - 3x + 6, x\right)$	□	□	Univariate function minimization
$\text{fMax}(x + 2 \cdot \sin(x), x) \mid x \geq -10 \text{ and } x \leq 10$	□	□	Univariate function maximization
$\sum\left(k^{10}, k, 0, n\right)$	1.94	□	Indefinite summation – polynomial
$\sum(2k^7 + 3k^5 - k - 1, k, 0, n)$	1.59	□	Indefinite summation – polynomial
$\text{Gosper}\left(1 \big/ \left(k^2 + \sqrt{5} \cdot k - 1\right), k, 0, n\right)$	□	□	Indefinite summation – rational function (ℜ6)
$\sum\left((-1)^k \cdot k \big/ \left(4k^2 - 1\right), k, 1, n\right)$	0.69	□	Indefinite summation – alternating rational function
$\sum\left(k^3 \cdot 2^k, k, 0, n\right)$	0.68	3.23	Indefinite summation – exponential ∗ polynomial
$\sum\left(a^{k/2+1/4} \cdot \text{poly4}, k, 1, n\right)$	□	□	Indefinite summation – exponential ∗ polynomial
$\text{Gosper}(k \cdot k!, k, 0, n)$	□	1.17	Indefinite summation – hypergeometric
$\text{Gosper}\left(4^k \big/ \text{nCr}(2k, k), k, 0, n - 1\right)$	□	□	Indefinite summation – hypergeometric
$\text{Gosper}\left(k^4 \cdot 4^k \big/ \text{nCr}(2k, k), k, 0, n\right)$	□	□	Indefinite summation – hypergeometric (ℜ7)
$\sum(1/i, i, 1, 400)$	5.42	□	Summation over a finite range
$\sum(1.0/i, i, 1, 400)$	2.32	□	Summation over a finite range (approximate)
$\sum\left(1 \big/ k^{42}, k, 1, \infty\right)$	4.42	11.87	Summation over an infinite range – Riemann zeta function
□	□	□	Summation over an infinite range
□	□	□	Numeric summation
det (mat1)	25.74	□	Matrix determinant
det (mat2)	49.42	□	Matrix determinant
mat1^{-1}	125.58	□	Matrix inverse
mat2^{-1}	167.95	□	Matrix inverse
mat1^{T}	0.68	□	Matrix transpose
mat2^{T}	1.41	□	Matrix transpose
eigVl (mat2)	98.11	□	Eigenvalues (numeric)
Eigenval (symbmat1)	36.95	□	Eigenvalues (symbolic)
□	□	□	LU decomposition (numeric)
□	□	□	QR decomposition (numeric)

□	□	□	Cholesky decomposition
simult (mat2, vec1)	□	□	Linear system of equations
□	□	□	Linear system of equations – multiple right-hand sides
rref (mat1)	47.42	□	Row-reduced echelon form
□	□	□	Row-reduced echelon form
e^{mat2}	824.53	□	Matrix exponential (should perhaps try diagonal flattening)
mat1^{10}	20.95	□	Matrix power (should perhaps try binary splitting)
mat2^{10}	237.21	□	Matrix power
mat6^{π}	157.63	□	Matrix power
SVD (mat3)	□	□	Singular value decomposition
PInverse (mat3)	□	□	Pseudoinverse
mat2 ∗ mat2	27.58	□	Matrix-matrix dot product
mat2 ∗ vec1	1.66	□	Matrix-vector dot product
□	□	□	Vector-vector dot product
□	□	□	Matrix norm (Frobenius)
Hilbert (60)	7.63	28.92	Hilbert matrix
Vanderm (symlist1)	7.26	□	Vandermonde matrix
FFT (list2)	22.68	39.74	Fast Fourier Transform
ChinRem ({{111, 853}, {4293, 283}})	0.05	0.14	Chinese remainder for integers
□	□	□	Extended gcd for integers
PowerMod (129140163, 488281255, 7)	0.03	0.11	Integer exponentiation modulo a prime
MultMod (123456789, 987654321, 7)	0.01	0.08	Integer multiplication modulo a prime
PowerMod (32361, −1, 113)	0.02	0.13	Modular inverse
NxtPrime (167168452281)	3.03	2.35	Next prime number
Totient (817723175441)	4.05	1.31	Euler totient function
Lagrange (mat5, x)	3.34	0.70	Lagrange interpolation
mean (list1)	0.09	□	Mean of a list
median (list1)	0.19	□	Median of a list
stdDev (list1)	0.30	□	Standard deviation of a list
□	□	□	Linear regression
Gamma (211 / 2)	1.57	No	Gamma function – half-integer arguments (exact)
Gamma (11.11)	1.03	0.03	Gamma function – real arguments
Gamma $(-3 + 2\,i)$	5.03	0.53	Gamma function – complex arguments
Psi (0, 71 / 2)	0.89	No	Polygamma function – digamma (exact)
Psi (0, 31 / 7)	2.65	No	Polygamma function – digamma (exact)
Psi (0, i)	1.47	0.38	Polygamma function – digamma
Psi (0, −111.11)	1.05	0.04	Polygamma function – digamma
Psi (13, 2)	1.24	1.05	Polygamma function (exact)
Psi (11, 11.11)	19.00	Error	Polygamma function
nCr (1000, 453)	4.34	15.63	Binomial coefficient function
nCr (100000, −3333)	□	□	Binomial coefficient function
BernNum (50)	6.46	14.74	Bernoulli numbers (exact)

BesselJ (51 / 2, x)	5.79	□	BesselJ function – half-integer order
BesselJ (6, 27.3)	3.79	□	BesselJ function – integer order
BesselJ (111 – 222 *i*, 0)	0.24	□	BesselJ function
ChebyT (123, 456)	2.32	22.93	ChebyshevT polynomials – integer order (exact)
ChebyT (22, x)	8.05	0.42	ChebyshevT polynomials – integer order
ChebyT (2.1, 3.2)	0.27	Error	ChebyshevT polynomials
Hermite (77, x)	8.89	6.42	Hermite polynomials – integer order
Hermite (100, 1)	15.95	14.2	Hermite polynomials – integer order (exact)
Hermite (3 / 4, 0.1)	20.79	Error	Hermite polynomials – fractional order
□	□	□	Hermite polynomials
Legendre (55, x)	6.35	16.17	Legendre polynomial (first kind)
□	□	□	Laguerre polynomials
□	□	□	Spherical harmonics

The following are computed separately and are not included in the timings, since they are just inputs.

Symbol	Value
num1	$3 * 11 * 6214223 * 337636146415297021$
list1	seq (rand (), i, 1, 40)
list2	seq (rand (), i, 1, 512)
mat1	randMat (15, 15)
mat2	seq (seq (rand (), j, 1, 20), i, 1, 20)
mat3	seq (seq (rand (), j, 1, 20), i, 1, 10)
mat4	seq ({i, prime (i)}, i, 1, 20)
mat5	{{2, 9}, {4, 833}, {6, 7129}, {8, 31233}, {10, 97001}, {12, 243649}}
mat6	seq (seq (rand (), j, 1, 8), i, 1, 8)
symbmat1	diag (seq (expr (string (x) & string (i)), i, 1, 15))
symlist1	seq (expr (string (x) & string (i)), i, 1, 30)
vec1	seq ({rand ()}, i, 1, 20)
poly1	$\text{expand}\left(3 * x * (5x - 7)^2 * \left(x^2 - 3x + 1\right) * \left(x^4 + x - 5\right)\right)$
poly2	$\text{expand}\left((3 * x^7 + y) * \left(x - y^2 - 1\right)\right)$
poly3	$4x^9 + 21x^6 + 2x^3 - 3x^2 - 3$
poly4	$k^4 - 3k^3 + 5k^2 - 8k + 7$
poly5	expand (∏ (x − i, i, 1, 10))
poly6	$x^8 - x^7 - 9 * x^6 + 3 * x^5 + 21 * x^4 - 3 * x^3 - 19 * x^2 + x + 6$
poly7	$12x^3 - 28x^2 + 20x - 4$
poly8	$-12x^2 + 10x - 2$
poly9	randPoly (50, x)
poly10	∏ (x − i, i, 1, 10)
polysys	$2 * x^2 + x * y - y + 1 = 0$ and $-3 * x * y - x + 2 * y^2 - 2 = 0$ and $2 * x^2 - 3 * x * y^2 + 2 * y^3 - 3 * y + 1 = 0$
ratfunc1	$\text{expand}\left(\left((x + y)^2 - (z + 1)^2\right)^n \middle/ (x + y + z + 1)^{2n} \middle\vert n = 3\right)$
ode1	$x \cdot y^2 \cdot (y')^2 - 2 \cdot y^3 \cdot y' + 2 \cdot x \cdot y^2 = x^3$
initcond1	$y(0) = 1$ and $y'(0) = 0$
boundcond1	$y(2) = 3$ and $y(5) = 2$

Graphics performance

■ **The 68k has the following 3D function plots:**

Wireframe

HiddenSurface

ContourLevels (2D contour plot)

WireAndContour

ImplicitPlot (2D implicit plot)

■ The 68k has the following differential equation plots:

SlopeField

DirectionField (can select axes from all dependent and independent variables)

FieldOff (solution plot; can select axes from all dependent and independent variables)

■ Sequence plots on the 68k:

Can have time, web, or custom axes

Can select axes from all dependent and independent variables

■ Conic plots:

The 49G+ has conic plots, but as far as I can tell, these are trivially replicated on the 68k by solving for y or z as appropriate (or doing a 2D implicit plot)

Example	TI	HP	Comments
$\sin(x)\cdot\cos(x),\ \{x,\ -2\pi,\ 2\pi\}$	□	□	2 D function plots – simple function
floor (x), {x, −4, 4}	□	□	2 D function plots – piecewise function
mod (ceiling (x), 3), {x, 0, 10}	□	□	2 D function plots – piecewise function
$\min(\sin(x), \cos(x)),\ \{x,\ -3\pi/2,\ 3\pi/2\}$	□	□	2 D function plots – piecewise function
$\text{when}\left(x > 7/3,\ x^{1/3},\ \sin(x^5)\right),\ \{x,\ -2,\ 2\}$	□	□	2 D function plots – piecewise function
piecewis1, {x, −3, 3}	□	□	2 D function plots – piecewise function
$\left(x^2 - x + 1\right)/\left((x-1)*x^2\right)$	□	□	2 D function plots – poles
tan (x), {x, −5, 5}	□	□	2 D function plots – simple poles
$x^2\ \text{when}\left(\sin\left(x^3\right) > 0,\ 1,\ 0\right),\ \{x,\ 0,\ 10\}$	□	□	2 D function plots – many discontinuities
$\sin(1/x),\ \{x,\ -1,\ 1\}$	□	□	2 D function plots – singularity
□	□	□	2 D parametric plots
□	□	□	2 D polar plots
□	□	□	2 D sequence plots
□	□	□	2 D implicit plots
expr1, {x, −3, 3}, {y, −3, 3}	□	□	2 D contour plots
$\frac{y}{x^2+y^2+1},\ \{x,\ -5,\ 5\},\ \{y,\ -5,\ 5\}$	□	□	3 D function plots
piecewis2, {x, −3, 3}, {y, −3, 3}	□	□	3 D function plots – piecewise function
$\text{real}\left(\tan^{-1}(x + i\,y)\right),\ \{x,\ -\pi,\ \pi\},\ \{y,\ -\pi,\ \pi\}$	□	□	3 D function plots – discontinuities
$\sin(100\,x\,y),\ \{x,\ -\pi,\ \pi\},\ \{y,\ -\pi,\ \pi\}$	□	□	3 D function plots – high frequencies
□	□	□	3 D surface rotation
□	□	□	Differential equation plots
□	□	□	Statistics scatter plots
□	□	□	Statistics bar chart plots
hist1	□	□	Statistics histogram plots

Symbol	Value
piecewis1	when (x < −1, −1, when (−1 ≤ x ≤ 1, x, when (x > 1, 1, undef)))
piecewis2	$\text{when}\left(x^2 + y^2 \le 2,\ x^2 + y^2 + 2,\ \sin\left(\left(x^2 + y^2\right)/2\right)\right)$
expr1	$\frac{1}{3}\,e^{-(y+1)^2 - x\,(x+2)}\left(9\,e^{2\,x}\,(x-1)^2 + e^{2\,y}\left(6\,e^{2\,x+1}\left(5\,y^5 + 5\,x^3 - x\right) - 1\right)\right)$
hist1	seq (randNorm (2.2, 0.71), i, 1, 50)

Display routines

The display routines on the 68k are considered rather poor for large expressions. Having many large expressions in the history can slow the home screen down a lot, and often influences computation time.

Example	TI	HP	Comments
299!	9	□	Big integers
seq (seq (rand (), j, 1, 40), i, 1, 40)	17	□	Large matrices
randMat (40, 40)	7	□	Large matrices
□	□	□	□
□	□	□	□
□	□	□	□

Functionality

Functionality	TI	HP	Comments
Converting to various trig / exp / hyperbolic forms	□	✓	The 49 G has a lot of expression rewriting tools
Cubic and quartic equations	✓	□	□
Root isolation	□	□	Finding a bounding rectangle for each root
Nth-order derivatives	✓	□	One could write a 49 G + program for this, but there would be significant performance and simplification issues
Nth-order implicit differentiation	✓	□	□
Limits of expressions involving factorials	✓	□	Example : limit (x! − x, x, ∞)
Univariate inequality solving	✓	□	Could use Groebner basis techniques for polynomial systems of inequalities
Polynomial roots modulo a prime	✓	□	□
Quadratic diophantine equations	✓	□	Would be nice to be able to handle Thue equations
Polynomial factorization over the integers	□	✓	Ability to recognize cyclotomic polynomials, factor expand $\left(\left(x^3+x+1\right)\left(x^3-x+1\right)\right)$
Squarefree factorization	✓	□	Sometimes we need only a fast partial factorization
Equation solving with units	✓	□	□
Continuous integrals for continuous integrands	✓	✓	Example : $\int \frac{1}{2\cos(x)+3}\,dx$
Rothstein-Trager algorithm	✓	□	Example : $\int \frac{6x^5+6x^4-8x^3-18x^2+8x+8}{x^6-5x^4-8x^3-2x^2+2x+1}\,dx$
Risch integration	□	✓	I attempted a (nearly useless) Risch implementation for the TI-68k
Rational function integration (high-degree)	✓	□	Example : $\int \left(1/\left(x^4+x^3+x^2+x+1\right)\right)dx$
Heuristic integration including table lookup	✓	□	Useful for speed and for cases not covered by algorithms
Hyperelliptic integrals	✓	□	Example : $\int\left(\frac{3x^5-x+2}{\sqrt{x^5+x-1}\,\left(x^5-x^2+x-1\right)}\right)dx$
Subresultants	✓	□	□
Reduction of a polynomial wrt other polynomials	✓	□	"Simplification" with side relations
Heuristic GCD	□	✓	GCDHEU on the 49 G, I believe

Solving simple difference equations	✓	□	Similar to solving differential equations
Integral transforms and inverses	✓	□	Fourier, Laplace (Hilbert, Mellin, Hankel not done yet)
Z-transform and inverse	✓	□	□
Multidimensional definite integrals	✓	□	□
Complex residues	✓	□	Would be helpful for contour integration
Symbolic Laurent series	□	✓	Including asymptotic series
Symbolic products	✓	□	□
Pade approximant	✓	□	□
Series arithmetic and series composition	□	□	□
Zero-equivalence testing	□	□	Pretty hard problem
Simplification	□	□	Example : $\sqrt{2\cdot\sqrt{3}+4} \rightarrow \sqrt{3}+1$
Vector analysis in general coordinate systems	✓	□	Conversion between systems missing from MathTools
Quine-McCluskey algorithm	✓	□	Minimal SOP or POS expression from minterm or maxterm lists
Tensor analysis	✓	□	Arrays package by Doug Burkett and me; focus on GR
Optimization	✓	□	Linear programming, Lagrange multipliers
Symbolic solution of ODEs	✓	□	Linear high-order constant-coefficient (non) homogeneous
Symbolic solution of PDEs	✓	□	Linear constant-coefficient PDEs, quasilinear first-order PDEs, separation of variables, etc.
Complete integral solutions for PDEs	□	□	□
5 th-order Runge-Kutta for numerical ODE solving	✓	□	□
Numerical ODE boundary value problem solving	□	□	□
Symbolic solution of systems of ODEs	✓	□	Lars Frederiksen's DiffEq package can do this; Lin1ODEs from MathTools can solve linear first-order homogeneous ODE systems
Integral equations	✓	□	Lars Frederiksen's Advanced Laplace package can do this
Statistics	✓	✓	Distributions / inverses, random numbers, regressions, hypothesis tests, (M) Anova, confidence intervals. Stats app on the 68 k, Stat49Pro on the 49 G + .
Nonlinear regression for arbitrary model function	□	□	SVD could be useful here
Numerical definite integration	□	□	Ability to choose method and precision of result
Continued fractions	✓	□	□
Binomial coefficients with negative numbers	✓	□	Examples : nCr $(-2, 3)$, nCr $(3, -2)$
Arbitrary-precision arithmetic	□	✓	There is a GMP demo for the TI-68k
Matrix operations modulo a prime	✓	□	Determinant, inverse, and nullspace (need to add power)
Solving singular linear systems	✓	□	TI's SimultEq Flash app
Kronecker product of matrices	✓	□	□
Gram-Schmidt orthonormalization	✓	□	I think the 49 G + also has this
Schur, LQ matrix decompositions	□	✓	□
Symbolic matrix decompositions	✓	□	□
Sparse matrices	□	□	MathTools has very limited sparse matrix support
Finding a linear recurrence for data	✓	□	□

Series convergence acceleration	✓	□	Aitken δ^2, Euler-Maclaurin
Radix conversions for fractional parts	✓	□	□
Permutations, subsets, etc. of a list	✓	□	□
Prime factorization for large numbers	✓	□	Example : 692388281206151078910795 39
Prime numbers	✓	□	nth prime, random prime in interval, number of primes ≤ n
Chinese remaindering	✓	□	Solving more than two congruences simultaneously
Lattice reduction using LLL	✓	□	Can be useful for factor recombination (polynomial factoring)
PSLQ integer relation algorithm	□	✓	Can be useful for number recognition
Algebraic number arithmetic	□	□	There is some basic stuff for this in MathTools
Domain membership checking	✓	□	Whether a numerical quantity is an element of a domain
Graph editing and drawing	□	✓	With additional routines to find shortest or cheapest path, do various sorts, etc.
Geometry drawing and transformations	✓	□	Sketchpad, Cabri
Import / export of standard data / graphics formats	✓	□	Excel spreadsheet, MathML, XML, etc.
Y-slice plots	□	✓	Animated plot of z (x, y) vs y for changing values of x
Statistics xy-line and modified box plots	✓	□	□

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