CAS comparison – TI89/HP49G

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All timings (the “TI89” & “HP49G” columns) are in seconds. The ROM on the HP49G is version v1.14.2, which is a beta release. This release seems to have some slowdowns in a few CAS commands compared to the official current v1.10 release, ranging from 2% to a factor 4 in one severe case (a non-rational integration that takes around 9 seconds on v1.10 and takes 36,55 seconds on this release)! Other operations are noticeably faster in this release.
There’s no timing facilities on the TI89, therefore all time measurements in regard to operations on that machine is done by external stopwatch – an average from three timings and rounded to nearest half a second is used.
Results that appear ‘instantaneous’ are written as “< 1”, and operations not finished in 1000 seconds are aborted and written as “> 1000”.

RATIONAL SIMPLIFICATIONS & FACTOR TESTS

Command: TI89 → FACTOR(,x)
HP49G → FACTOR, VX: x

Expression	Result	TI89	HP49G
$x^4+7\cdot x^3-23\cdot x^2-93\cdot x+36$	$(x+3)\cdot(x-4)\cdot\left(x^2+8\cdot x-3\right)$	3,50	1,47
$6\cdot x^3-2\cdot x^2+21\cdot x-7$	$(3\cdot x-1)\cdot\left(2\cdot x^2+7\right)$	2,00	0,95
$2\cdot x^3+y\cdot x^2-\left(2\cdot y^2-3\cdot y\right)\cdot x-2\cdot y^2$	$(3\cdot x-2\cdot y)\cdot\left(x^2+x\cdot y+y\right)$	3,00	2,91
$(y-z)\cdot x^2-\left(y^2-z^2\right)\cdot x+z\cdot y^2-z^2\cdot y$	$(x-y)\cdot(x-z)\cdot(y-z)$	1,50	2,79
$(y+2)\cdot x^2+\left[(z-3)\cdot y^2+(2\cdot z-6)\cdot y-z\right]\cdot x-\left(z^2-3\cdot z\right)\cdot y$	$(x\cdot y+2\cdot x-z)\cdot(x+y\cdot z-3\cdot y)$	4,00	3,01
$x^5-2\cdot y\cdot x^4+3\cdot y^2\cdot x^3-3\cdot y^3\cdot x^2+2\cdot y^4\cdot x-y^5$	$(x-y)\cdot\left(x^2-x\cdot y+y^2\right)\cdot\left(x^2+y^2\right)$	3,50	5,49
$x^{10}-19\cdot x^9+138\cdot x^8-422\cdot x^7+41\cdot x^6+3093\cdot x^5-6788\cdot x^4+356\cdot x^3+15696\cdot x^2-19008\cdot x+6912$	$(x-1)^2\cdot(x+2)^2\cdot(x-3)^3\cdot(x-4)^3$	6,50	3,38
$y\cdot x\cdot a^2-(y\cdot b+z\cdot x\cdot c)\cdot a+z\cdot c\cdot b$	$(a\cdot x-b)\cdot(a\cdot y-c\cdot z)$	< 1	4,46
$4096\cdot x^8-(14336\cdot y-43008)\cdot x^7+\left(16768\cdot y^2-155904\cdot y+169344\right)\cdot x^6-\left(5600\cdot y^3-195552\cdot y^2+635040\cdot y-296352\right)\cdot x^5-\left(1919\cdot y^4+83244\cdot y^3-849366\cdot y^2+1148364\cdot y-194481\right)\cdot x^4+\left(700\cdot y^5-9744\cdot y^4-433944\cdot y^3+1629936\cdot y^2-777924\cdot y\right)\cdot x^3+\left(262\cdot y^6+8568\cdot y^5+15876\cdot y^4-963144\cdot y^3+1166886\cdot y^2\right)\cdot x^2+\left(28\cdot y^7+1680\cdot y^6+31752\cdot y^5+148176\cdot y^4-777924\cdot y^3\right)\cdot x+y^8+84\cdot y^7+2646\cdot y^6+37044\cdot y^5+194481\cdot y^4$	$(x-y)^4\cdot(8\cdot x+y+21)^4$	86,00	12,92
$a^2-(2\cdot x+1)\cdot a+x^2+x$	$(a-x)\cdot(a-x-1)$	1,00	1,19
$x^6+2\cdot x^3-x^2+1$	$\left(x^3+x+1\right)\cdot\left(x^3-x+1\right)$	Fail	5,94
$x^8-30\cdot x^6+273\cdot x^4-820\cdot x^2+576$	$(x-1)\cdot(x-2)\cdot(x-3)\cdot(x-4)\cdot(x+1)\cdot(x+2)\cdot(x+3)\cdot(x+4)$	3,50	3,92
$120\cdot x^8-1526\cdot x^7+7789\cdot x^6-20846\cdot x^5+32179\cdot x^4-29624\cdot x^3+16036\cdot x^2-4704\cdot x+576$	$(x-1)\cdot(x-2)\cdot(x-3)\cdot(x-4)\cdot(2\cdot x-1)\cdot(3\cdot x-2)\cdot(4\cdot x-3)\cdot(5\cdot x-4)$	16,00	6,05
$x^3-3\cdot y\cdot x^2+\left(3\cdot y^2-1\right)\cdot x-\left(y^3-y\right)$	$(x-y+1)\cdot(x-y)\cdot(x-y-1)$	4,50	2,31
$x^4-2\cdot x^3-x^2-2\cdot x+1$	$\left(x^2-3\cdot x+1\right)\cdot\left(x^2+x+1\right)$	2,00	1,63
$x^4-140\cdot x^3+7290\cdot x^2-167380\cdot x+1430309$	$(x-29)\cdot(x-31)\cdot(x-37)\cdot(x-43)$	2,00	2,56
$x^3-3000\cdot x^2+2999999\cdot x-999999000$	$(x-999)\cdot(x-1000)\cdot(x-1001)$	5,00	1,87
$a^3-3\cdot c\cdot b\cdot a+b^3+c^3$	$(a+b+c)$	3,50	4,27

	$\cdot\left(a^2 - a\cdot b + b^2 - a\cdot c - b\cdot c + c^2\right)$		
$x^3 - (y - z)\cdot x^2 - \left(y^2 + 2\cdot z\cdot y + z^2\right)\cdot x + y^3 + z\cdot y^2 - z^2\cdot y - z^3$	$(x - y - z)\cdot(x - y + z)\cdot(x + y + z)$	12,00	4,81
$(3\cdot y + 9)\cdot x^2 - \left(y^3 + 5\cdot y^2 + 2\cdot y + 18\right)\cdot x + y^4 - y^3 + 6\cdot y^2 - y + 5$	$\left(x\cdot y + 3\cdot x - y^2 - 1\right)\cdot\left(3\cdot x - y^2 + y - 5\right)$	6,00	1,71
$(6\cdot y + 12)\cdot x^5 + \left[(6\cdot z - 18)\cdot y^2 + (12\cdot z - 36)\cdot y - 6\cdot z\right]\cdot x^4$ $+ \left[y^2 - \left(6\cdot z^2 - 19\cdot z - 2\right)\cdot y + 2\cdot z\right]\cdot x^3$ $+ \left[(z - 3)\cdot y^3 + \left(z^2 - z - 6\right)\cdot y^2 + \left(2\cdot z^2 - 7\cdot z + 1\right)\cdot y - \left(z^2 - 2\right)\right]\cdot x^2$ $- \left[\left(z^2 - 4\cdot z + 3\right)\cdot y^2 + \left(z^3 - 3\cdot z^2 - 2\cdot z + 6\right)\cdot y + z\right]\cdot x - \left(z^2 - 3\cdot z\right)\cdot y$	$(x\cdot y + 2\cdot x - z)\cdot(x + y\cdot z - 3\cdot y)$ $\cdot\left(6\cdot x^3 + x\cdot y + x\cdot z + 1\right)$	Fail	16,83
$6\cdot x^6 - \left[126\cdot z\cdot y^3 - \left(78\cdot z^2 + 1\right)\cdot y - z\right]\cdot x^4 + 13\cdot x^3$ $- \left(21\cdot z\cdot y^4 + 21\cdot z^2\cdot y^3 - 13\cdot z^2\cdot y^2 - 13\cdot z^3\cdot y\right)\cdot x^2$ $- \left[21\cdot z\cdot y^3 - \left(13\cdot z^2 + 2\right)\cdot y - 2\cdot z\right]\cdot x + 2$	$\left(x^3 - 21\cdot x\cdot z\cdot y^3 + 13\cdot x\cdot y\cdot z^2 + 2\right)$ $\cdot\left(6\cdot x^3 + x\cdot y + x\cdot z + 1\right)$	Fail	58,63
$z^5\cdot y^2\cdot x^4 + z^6\cdot y^2\cdot x^3 - \left(z^5\cdot y^4 + z^6\cdot y^3\right)\cdot x^2$	$x^2\cdot y^2\cdot z^5\cdot(x - y)\cdot(x + y + z)$	1,00	2,78
$z\cdot y^2\cdot x^6 - \left(z\cdot y^3 - z^2\cdot y^2\right)\cdot x^5 - \left(z\cdot y^4 + 2\cdot z^2\cdot y^3 + z^3\cdot y^2\right)\cdot x^4$ $+ \left(z\cdot y^5 + z^2\cdot y^4 - z^3\cdot y^3 - z^4\cdot y^2\right)\cdot x^3$	$x^3\cdot y^2\cdot z\cdot(x - y - z)\cdot(x - y + z)\cdot(x + y + z)$	14,00	5,95
$25920000\cdot x^6 - 25920000\cdot x^5 - 122016600\cdot x^4 + 157818150\cdot x^3$ $+ 29607617\cdot x^2 - 58291959\cdot x - 7411976$	$(120\cdot x - 209)\cdot(30\cdot x - 31)\cdot(12\cdot x - 13)$ $\cdot(8\cdot x + 1)\cdot(15\cdot x + 8)\cdot(5\cdot x + 11)$	130,00	8,82
$x^{40} + x^{30} + x^{20} + x^{10} + 1$	$\left(x^{20} + x^{15} + x^{10} + x^5 + 1\right)$ $\cdot\left(x^{20} - x^{15} + x^{10} - x^5 + 1\right)$	> 1000,00	224,22

Notes:

- The GCD implementation on the TI89 is very slow as soon as multivariate fractions are to be simplified. With 3 or more variables, you have most of the time failures (user abort). The last expression running in more than 15 minutes is an example of a result not achieved because of a user abort.
- The factorization on the TI89 does not find the full factorization over the integers (Berlekamp algorithm not implemented). This is what causes some factorizations to Fail. The expression is then returned unfactored
- The HP49G also features a complex mode to allow complex partial fractions (with PARTFRAC). This is useful when preparing z-transforms and the like. There is no such equivalent on the TI89, as cExpand isn't implemented (Expand is used for partial fractions on the TI89).

The HP49G has a better factorization implementation and makes more factorizations. When there is a large difference in the timings of the two machines, it's the '49G that's in front by a large amount. A clear win for the HP.

RATIONAL PARTIAL FRACTION TESTS

Command: TI89 → EXPAND(,x)
HP49G → PARTFRAC, VX: x

Expression	Result	TI89	HP49G
$\dfrac{2\cdot x - 1}{x^2 - 7\cdot x + 12}$	$\dfrac{7}{x - 4} - \dfrac{5}{x - 3}$	< 1	0,96
$\dfrac{1}{x^4 - 1}$	$\dfrac{\frac{1}{4}}{x - 1} - \dfrac{\frac{1}{4}}{x + 1} - \dfrac{\frac{1}{2}}{x^2 + 1}$	1,00	1,47

$\frac{1}{x^4+1}$	$\frac{\frac{\sqrt{2}\cdot x+2}{4}}{x^2+\sqrt{2}\cdot x+1}-\frac{\frac{\sqrt{2}\cdot x-2}{4}}{x^2-\sqrt{2}\cdot x+1}$	1,00	9,72
$\frac{1}{\left(x^4-1\right)^4}$	$\frac{\frac{1}{256}}{(x-1)^4}-\frac{\frac{3}{128}}{(x-1)^3}+\frac{\frac{37}{512}}{(x-1)^2}-\frac{\frac{77}{512}}{x-1}+\frac{\frac{1}{256}}{(x+1)^4}+\frac{\frac{3}{128}}{(x+1)^3}$ $+\frac{\frac{37}{512}}{(x+1)^2}+\frac{\frac{77}{512}}{x+1}+\frac{\frac{1}{16}}{\left(x^2+1\right)^4}+\frac{\frac{1}{8}}{\left(x^2+1\right)^3}+\frac{\frac{5}{32}}{\left(x^2+1\right)^2}+\frac{\frac{5}{32}}{x^2+1}$	5,00	6,62
$-\frac{x^3-x^2-2\cdot x+1}{x^5-x^4-3\cdot x^3+3\cdot x^2}$	$\frac{1}{3\cdot x}-\frac{1}{3\cdot x^2}-\frac{1}{2\cdot(x-1)}+\frac{x-1}{6\cdot\left(x^2-3\right)}$	2,00	3,24
$-\frac{(2\cdot y-7)\cdot x-\left(9\cdot y^2+21\cdot y+10\right)}{y\cdot x^2+\left(3\cdot y^2-2\right)\cdot x-6\cdot y}$	$\frac{3\cdot y+7}{x\cdot y-2}-\frac{5}{x+3\cdot y}$	3,00	2,83
$\frac{1}{3\cdot x\cdot\left(x^2+x+1\right)\cdot\left(x^2-3\cdot x+1\right)\cdot(x-5)^2}$	$\frac{\frac{1}{75}}{x}-\frac{\frac{677}{290725}}{x-5}+\frac{\frac{1}{5115}}{(x-5)^2}-\frac{\frac{2}{961}\cdot x}{x^2+x+1}+\frac{\frac{11}{11532}}{x^2+x+1}-\frac{\frac{4}{363}\cdot x}{x^2-3\cdot x+1}+\frac{\frac{15}{484}}{x^2-3\cdot x+1}$	3,00	8,35
$\frac{\left(y^2-y\right)\cdot x^4+\left(2\cdot y^3-3\cdot y^2+3\cdot y-1\right)\cdot x^3}{\left(y^2-y\right)\cdot x^3+\left(y^3-2\cdot y^2+y\right)\cdot x^2-\left(y^3-y^2\right)\cdot x}$ $+\frac{\left(y^4-3\cdot y^3+7\cdot y^2-6\cdot y+1\right)\cdot x^2}{\left(y^2-y\right)\cdot x^3+\left(y^3-2\cdot y^2+y\right)\cdot x^2-\left(y^3-y^2\right)\cdot x}$ $-\frac{\left(y^4-3\cdot y^3+6\cdot y^2-3\cdot y\right)\cdot x+\left(y^3-y^2\right)}{\left(y^2-y\right)\cdot x^3+\left(y^3-2\cdot y^2+y\right)\cdot x^2-\left(y^3-y^2\right)\cdot x}$	$x+y+\frac{1}{y}+\frac{1}{y-1}+\frac{1}{x}+\frac{1}{x-1}+\frac{1}{x+y}$	9,00	6,55
$\frac{1}{x^2-y^2}$	$\frac{\frac{1}{2\cdot y}}{x-y}-\frac{\frac{1}{2\cdot y}}{x+y}$	< 1	1,58
$\frac{1}{y\cdot x+y^2}$	$\frac{\frac{1}{y}}{x+y}$	< 1	0,72
$\frac{x^{10}}{(x-1)^2\cdot\left(x^2+x+1\right)\cdot\left(x^2+1\right)^3}$	$1+\frac{\frac{1}{24}}{(x-1)^2}+\frac{\frac{1}{4}}{x-1}+\frac{\frac{1}{3}}{x^2+x+1}-\frac{\frac{1}{2}}{\left(x^2+1\right)^3}-\frac{\frac{x-8}{4}}{\left(x^2+1\right)^2}+\frac{\frac{6\cdot x-21}{8}}{x^2+1}$	4,50	3,80

In the field of real partial fractions, it seems that the TI89 has a slight edge over the HP, but where the HP is fastest, it often is by a large margin. That makes this a tie.
When it comes to complex partial fractions (not covered here) the HP49G blows the opposition away though. This is not implemented on the TI at all, and none of the usual complex handling functions (cFactor & cSolve) can be made to handle this in an orderly fashion.

RATIONAL FRACTION INTEGRATION TESTS

Command: TI89 → ∫(,x)
HP49G → INTVX, VX: x

Expression	Result	TI89	HP49G
$\frac{2\cdot x-1}{x^2-7\cdot x+12}$	$7\cdot\ln(x-4)-5\cdot\ln(x-3)$	< 1	1,79

$\frac{1}{x^4-1}$	$\frac{1}{4}\cdot\ln(x-1)-\frac{1}{4}\cdot\ln(x+1)-\frac{1}{2}\cdot\operatorname{atan}(x)$	1,00	2,16
$\frac{1}{x^4+1}$	$\frac{\sqrt{2}}{8}\cdot\ln\left(x^2+\sqrt{2}\cdot x+1\right)-\frac{\sqrt{2}}{8}\cdot\ln\left(x^2-\sqrt{2}\cdot x+1\right)+\frac{\sqrt{2}}{4}\cdot\operatorname{atan}\left(\sqrt{2}\cdot x-1\right)+\frac{\sqrt{2}}{4}\cdot\operatorname{atan}\left(\sqrt{2}\cdot x+1\right)$	4,50	5,91
$\frac{1}{\left(x^4-1\right)^4}$	$\frac{77}{256}\cdot\operatorname{atan}(x)+\frac{77}{512}\cdot\ln(x+1)-\frac{77}{512}\cdot\ln(x-1)-\frac{\frac{111\cdot x^2-240\cdot x+131}{1536}}{(x-1)^3}-\frac{\frac{111\cdot x^2+240\cdot x+131}{1536}}{(x+1)^3}+\frac{\frac{111\cdot x^5+256\cdot x^3+153\cdot x}{768}}{\left(x^2+1\right)^3}$	35,00	9,13
$\frac{1}{\left(x^4+1\right)^4}$	$\frac{\frac{77\cdot x^9+198\cdot x^5+153\cdot x}{384}}{\left(x^4+1\right)^3}-\frac{77\cdot\sqrt{2}}{1024}\cdot\ln\left(x^2-\sqrt{2}\cdot x+1\right)+\frac{77\cdot\sqrt{2}}{512}\cdot\operatorname{atan}\left(\sqrt{2}\cdot x-1\right)+\frac{77\cdot\sqrt{2}}{1024}\cdot\ln\left(x^2+\sqrt{2}\cdot x+1\right)+\frac{77\cdot\sqrt{2}}{512}\cdot\operatorname{atan}\left(\sqrt{2}\cdot x+1\right)$	130,00	8,47
$-\frac{x^3-x^2-2\cdot x+1}{x^5-x^4-3\cdot x^3+3\cdot x^2}$	$\frac{1}{12}\cdot\ln\left(x^2-3\right)+\frac{\sqrt{3}}{36}\cdot\ln\left(\frac{x+\sqrt{3}}{x-\sqrt{3}}\right)+\frac{1}{3}\cdot\ln(x)-\frac{1}{2}\cdot\ln(x-1)+\frac{\frac{1}{3}}{x}$	3,00	5,77
$-\frac{(2\cdot y-7)\cdot x-\left(9\cdot y^2+21\cdot y+10\right)}{y\cdot x^2+\left(3\cdot y^2-2\right)\cdot x-6\cdot y}$	$-\frac{3\cdot y+7}{y}\cdot\ln\left(\frac{y}{y\cdot x-2}\right)-5\cdot\ln(x+3\cdot y)$	6,00	5,80
$\frac{1}{3\cdot x\cdot\left(x^2+x+1\right)\cdot\left(x^2-3\cdot x+1\right)\cdot(x-5)^2}$	$\frac{1}{75}\cdot\ln(x)-\frac{677}{2907025}\cdot\ln(x-5)+\frac{23\cdot\sqrt{3}}{17298}\cdot\operatorname{atan}\left(\frac{2\cdot\sqrt{3}\cdot x+\sqrt{3}}{3}\right)-\frac{1}{961}\cdot\ln\left(x^2+x+1\right)-\frac{7\cdot\sqrt{5}}{2420}\cdot\ln\left[\frac{2\cdot x-\left(3-\sqrt{5}\right)}{2\cdot x-\left(3+\sqrt{5}\right)}\right]-\frac{2}{363}\cdot\ln\left(x^2-3\cdot x+1\right)-\frac{\frac{1}{5115}}{x-5}$	12,50	9,93
$\frac{\left(y^2-y\right)\cdot x^4+\left(2\cdot y^3-3\cdot y^2+3\cdot y-1\right)\cdot x^3}{\left(y^2-y\right)\cdot x^3+\left(y^3-2\cdot y^2+y\right)\cdot x^2-\left(y^3-y^2\right)\cdot x}+\frac{\left(y^4-3\cdot y^3+7\cdot y^2-6\cdot y+1\right)\cdot x^2}{\left(y^2-y\right)\cdot x^3+\left(y^3-2\cdot y^2+y\right)\cdot x^2-\left(y^3-y^2\right)\cdot x}-\frac{\left(y^4-3\cdot y^3+6\cdot y^2-3\cdot y\right)\cdot x+\left(y^3-y^2\right)}{\left(y^2-y\right)\cdot x^3+\left(y^3-2\cdot y^2+y\right)\cdot x^2-\left(y^3-y^2\right)\cdot x}$	$\frac{1}{2}\cdot x^2+\frac{y^3-y^2+2\cdot y-1}{y^2-y}\cdot x+\ln(x\cdot(x-1)\cdot(x+y))$	44,00	9,70
$\frac{1}{x^2-y^2}$	$-\frac{\ln\left(\frac{x+y}{x-y}\right)}{2\cdot y}$	< 1	2,51
$\frac{1}{y\cdot x+y^2}$	$\frac{\ln(x+y)}{y}$	< 1	1,36
$\frac{x^{10}}{(x-1)^2\cdot\left(x^2+x+1\right)\cdot\left(x^2+1\right)^3}$	$x-\frac{29}{16}\cdot\operatorname{atan}(x)+\frac{3}{8}\cdot\ln\left(x^2+1\right)+\frac{2\cdot\sqrt{3}}{9}\cdot\operatorname{atan}\left(\frac{2\cdot\sqrt{3}\cdot x+\sqrt{3}}{3}\right)+\frac{1}{4}\cdot\ln(x-1)-\frac{\frac{1}{24}}{x-1}+\frac{\frac{13\cdot x^3+2\cdot x^2+11\cdot x+2}{16}}{\left(x^2+1\right)^2}$	7,50	6,52

Here again it looks equal. A lot of the rational integrations the TI won, are done almost instantly, but half of the rational integrations the HP won was more than a factor three faster than the TI. This insinuates that the table–

based Derive engine of the TI89 has poor implementations of rational integration algorithms. So a tie it is, but with a edge pro the HP49G carried on to the next round.

NON-RATIONAL INTEGRATION TESTS

Command: TI89 → ∫(,x)
HP49G → INTVX, VX: x

Expression	Result	TI89	HP49G
$\frac{1}{(\sin(x)-2)^3}$	$-\frac{\sqrt{3}}{3}\cdot\operatorname{atan}\left(\frac{2\cdot\sqrt{3}\cdot\tan\left(\frac{x}{2}\right)-\sqrt{3}}{3}\right) - \frac{\frac{6\cdot\tan\left(\frac{x}{2}\right)^3-15\cdot\tan\left(\frac{x}{2}\right)^2+14\cdot\tan\left(\frac{x}{2}\right)-10}{24}}{\left(\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)+1\right)^2} - \frac{\sqrt{3}}{3}\cdot\pi\cdot\mathrm{FLOOR}\left(\frac{\frac{x}{2}}{\pi}+\frac{1}{2}\right)$	48,00	9,66
$x^8\cdot(1+\ln(x))^4$	$x^9\cdot\left(\frac{1}{9}\cdot\ln(x)^4+\frac{32}{81}\cdot\ln(x)^3+\frac{130}{243}\cdot\ln(x)^2+\frac{712}{2187}\cdot\ln(x)+\frac{1475}{19683}\right)$	2,00	8,00
$x^{10}\cdot\exp(2\cdot x)$	$\frac{4\cdot x^{10}-20\cdot x^9+90\cdot x^8-360\cdot x^7+1260\cdot x^6-3780\cdot x^5}{8}\cdot\exp(2\cdot x) + \frac{9450\cdot x^4-18900\cdot x^3+28350\cdot x^2-28350\cdot x+14175}{8}\cdot\exp(2\cdot x)$	2,00	3,62
$\left(2\cdot x^2+1\right)\cdot\exp\left(x^2\right)$	$x\cdot\exp\left(x^2\right)$	Fail	5,66
$\cos(8\cdot x)\cdot\cos(x)^8$	$\frac{x}{256}+\frac{1}{4096}\cdot\sin(16\cdot x)+\frac{1}{64}\cdot\sin(2\cdot x)+\frac{1}{448}\cdot\sin(14\cdot x)+\frac{7}{256}\cdot\sin(4\cdot x) + \frac{7}{768}\cdot\sin(12\cdot x)+\frac{7}{192}\cdot\sin(6\cdot x)+\frac{7}{320}\cdot\sin(10\cdot x)+\frac{35}{1024}\cdot\sin(8\cdot x)$	7,00	12,98
$\exp(x)\cdot\cos(x)^4\cdot x^3$	$\frac{3\cdot x^3-9\cdot x^2+18\cdot x-18}{8}\cdot\exp(x)+\frac{125\cdot x^3-150\cdot x^2-30\cdot x+72}{625}\cdot\sin(2\cdot x)\cdot\exp(x) + \frac{125\cdot x^3-150\cdot x^2-30\cdot x+72}{625}\cdot\sin(2\cdot x)\cdot\exp(x) + \frac{4913\cdot x^3+13005\cdot x^2-4794\cdot x-966}{668168}\cdot\cos(4\cdot x)\cdot\exp(x) + \frac{4913\cdot x^3-1734\cdot x^2-1326\cdot x+360}{167042}\cdot\sin(4\cdot x)\cdot\exp(x)$	55,00	9,79
$\frac{1}{1+\sqrt{x^2-1}}$	$\frac{\sqrt{2}}{4}\cdot\left[\ln\left(\frac{x+\sqrt{2}}{x-\sqrt{2}}\right)+\ln\left[\frac{x-\sqrt{2\cdot\left(x^2-1\right)}}{\left(1+\sqrt{2}\right)\cdot x+2+\sqrt{2}}\right]-\ln\left[-\frac{x+\sqrt{2\cdot\left(x^2-1\right)}}{\left(\sqrt{2}-1\right)\cdot x+2-\sqrt{2}}\right]\right] - \ln\left(\sqrt{x^2-1}-x\right)$	Fail	36,55
$x^4\cdot\sqrt{7+x-6\cdot x^2}$	$\frac{1105920\cdot x^5-18432\cdot x^4-326016\cdot x^3-92064\cdot x^2-589708\cdot x-362243}{6635520}\cdot\sqrt{-\left(6\cdot x^2-x-7\right)} + \operatorname{asin}\left(\frac{12\cdot x-1}{13}\right)\cdot\frac{\frac{1723631}{2654208}}{\sqrt{6}}$	13,00	13,45
$\frac{1}{(\cosh(x)-1)^4}$	$-\frac{\frac{140\cdot\exp(x)^3-84\cdot\exp(x)^2+28\cdot\exp(x)-4}{35}}{(\exp(x)-1)^7}$	4,00	7,16

$\frac{(x^2-1)\cdot\ln(x^2-1)-2\cdot x^2}{(x^2-1)\cdot\ln(x^2-1)^2}$	$\frac{x}{\ln(x^2-1)}$	Fail	12,66
$\frac{x\cdot\exp(x)^2-(x\cdot\ln(x)^2-2\cdot\ln(x)-x)\cdot\exp(x)+2\cdot\ln(x)}{x\cdot\exp(x)^2+2\cdot x\cdot\exp(x)+x}$	$\frac{\ln(x)^2}{\exp(x)+1}+\ln(\exp(x)+1)$	Fail	18,49

Notes:

- The Erable engine now covers probably all of the cases covered by the TI89 (sometimes slightly slower, sometimes slightly faster). On the other hand, many examples handled by Erable are not handled by the TI89:
  - Examples where the Risch algorithm is used like in #4, #10 & #11 above, general forms like X*EXP(X)/(X+1)^2, ... and so on
  - Examples where a square root of a 2nd order polynomial appears in the denominator of a rational fraction like in #7.
  - XROOT of a linear argument or fraction of linear arguments like XROOT(3,x+1).
- It seems that on the HP v1.14.2 is a little bit slower than v1.10 on some examples because of added tests for inputs of the form f(u)*u'.
- For trigonometric rational fractions, it seems to us that the TI89 use the complex change of variable y=exp(ix) instead of the real change u=tan(x/2) used by Erable. This assumption comes from the following observation: the TI89 is very fast for '1/(SIN(X)-1)^4' and very slow for '1/(SIN(X)+1)'. Partial fraction decomposition is made *before* change of variable, this could be implemented in Erable too and would make *pfexpaflag* unnecessary.

This test is pretty interesting, as it shows the TI89 hard at work. The general conception of the TI89 as a “lightning integrator” vaporizes here. The Derive engine is hard pressed to come up with results – those conceived faster than the HP are generally not by a large margin, where the other way round it is. In addition to this is the fact that when the TI89 fails, you have no option as to continue you work. Here the HP has a variety of tools to go the extra mile: various different direct integration tools (INT, INTVX and RISCH) and various advanced tools to continue you work when Erable itself runs out of ideas (e.g. integration by parts (IBP) and change of variable (SUBST)).
The question remains as to if you need these types of “advanced” integration routines. Are you never integrating more advanced expressions than rational fractions, then it’s a tie. Are your tool required to do more than just basic text-book examples, then there’s no way around the HP. Considering the slight edge of the HP in the rational fraction integration test, this is a clear win for the HP49G. Surprising to some, maybe, but a win nevertheless.

LIMIT/SERIES TESTS

Expression	Command	Result	Remarks	TI89	HP49G
$\lim_{x\to 0}\frac{\sin(x)}{x}$	LIMIT	1	Trivial undetermination	< 1	0,66
$\lim_{x\to\infty^+}\frac{\ln(x)}{x}$	LIMIT	0	Trivial undetermination	< 1	6,09
$\lim_{x\to 0}\frac{\sin(x)-x}{x^3}$	LIMIT	$-\frac{1}{6}$	Series expansion at 0	1,00	5,83
$\lim_{x\to\infty}\sin\left(x+\frac{1}{x}\right)-\sin(x)$	LIMIT	0	Trigonometric expansion	Fail	14,32
$\lim_{x\to 0^+}\frac{\ln(x^2+x)-\ln(x)-x}{x^2}$	LIMIT	$-\frac{1}{2}$	Asymptotic series	Fail	12,32

$\lim_{x\to\infty} \frac{\exp\left(\frac{x\cdot\exp(-x)}{\exp(-x)+\exp\left(\frac{-2\cdot x^2}{x+1}\right)}\right)-\exp(x)}{x}$	LIMIT	$-e^2$	A classical example of non-rational limit (From MuPad documentation)	Fail	63,14
$\exp(\operatorname{atan}(x))$	'X=0' 4 SERIES	$-\frac{7}{24}\cdot x^4-\frac{1}{6}\cdot x^3+\frac{1}{2}\cdot x^2+x+1$	A classical exercise	3,50	6,99
$\sin(\sinh(x))-\sinh(\sin(x))$	'X=0' 6 SERIES	0	First non-0 term at order 7	90,00	34,11
$\frac{\sin(x)}{\exp(x)-1}$	'X=0' 4 SERIES	$\frac{1}{12}\cdot x^3-\frac{1}{12}\cdot x^2-\frac{1}{2}\cdot x+1$	Undetermined form	Fail	6,94
x^x	'X=+0' 4 SERIES	$\frac{\ln(x)^4}{24}\cdot x^4+\frac{\ln(x)^3}{6}\cdot x^3+\frac{\ln(x)^2}{2}\cdot x^2+\ln(x)\cdot x+1$	Expansion w.r.t. $\ln(x)$	Fail	12,29
$\sqrt{x^2+x+1}$	'X=+∞' 4 SERIES	$\frac{2\cdot x+1}{2}+\frac{\frac{3}{128}}{x^3}-\frac{\frac{3}{16}}{x^2}+\frac{\frac{3}{8}}{x}$	Asymptotic expansion at infinity	Fail	10,23
$\operatorname{asin}(x)$	'X=-1' 4 SERIES	$\frac{35}{9216\cdot\sqrt{2\cdot x+2}}\cdot(x+1)^5+\frac{5}{448\cdot\sqrt{2\cdot x+2}}\cdot(x+1)^4+\frac{3}{80\cdot\sqrt{2\cdot x+2}}\cdot(x+1)^3+\frac{1}{6\cdot\sqrt{2\cdot x+2}}\cdot(x+1)^2+\frac{2}{\sqrt{2\cdot x+2}}\cdot(x+1)-\frac{\pi}{2}$	Expansion w.r.t. $\sqrt{x-1}$	Fail	28,27

Notes:

- The TI89 hasn't got a dedicated SERIES command. In the examples above series expansions was done with the TAYLOR command instead on that machine. That command has some weird quirks though, i.e. the very first series expansion was returned by the TI including some 'ln(e)' terms. These terms could not be cancelled out by any built in command (including EXPAND and FACTOR) while staying strictly symbolic, and had to be removed manually in the command line. Moreover, the TAYLOR and LIMIT commands of the TI89 seem to struggle already at relatively easy expressions; the first failure of the TI89 (the trigonometric expansion limit) returned 'undef' instead of an unevaluated answer. That's a genuine CAS error!
- In contrast to the above mentioned weaknesses of the TI89, the HP49G's SERIES command returns not only the expansion, but also the limit in the development point and the residual (error) term. The LIMIT command of the HP can compute both uni- and bi-directional limits.

The HP49G won this one hands down! Taylor expansion is a capability of the TI89, general series expansion really isn't.

SYMBOLIC MATRIX TESTS

For the symbolic matrix test, the following matrices were used;

Mat1 = 5x5 symbolic matrix:

$$\begin{bmatrix} x-1 & -2 & 0 & -4 & 0 \\ -5 & x-3 & -4 & 0 & -6 \\ 0 & -2 & x-5 & -6 & -2 \\ 0 & -3 & -2 & x+1 & 1 \\ 5 & -3 & -1 & 2 & x-8 \end{bmatrix}$$

Mat2 = 5x5 real matrix:

$$\begin{bmatrix} 1 & 2 & 0 & 4 & 0 \\ 5 & 3 & 4 & 0 & 6 \\ 0 & 2 & 5 & 6 & 2 \\ 0 & 3 & 2 & -1 & -1 \\ -5 & 3 & 1 & -2 & 8 \end{bmatrix}$$

Mat3 = 3x3 symbolic matrix:

$$\begin{bmatrix} 1 & 1 & A \\ 1 & A & 1 \\ A & 1 & 1 \end{bmatrix}$$

Mat4 = 12x12 real matrix:

$$\begin{bmatrix} -5 & -2 & 2 & 0 & 1 & 3 & -5 & -7 & -9 & 2 & 5 & 0 \\ -4 & 9 & 4 & -9 & -5 & 8 & -8 & 0 & 0 & 2 & 7 & -8 \\ -8 & 8 & 8 & 6 & 5 & -2 & 4 & 5 & 0 & 4 & -4 & 6 \\ -6 & -8 & -6 & -6 & 9 & 7 & -5 & 0 & 0 & 3 & 5 & -6 \\ -4 & -3 & -8 & -3 & 4 & 7 & 8 & 6 & 5 & -1 & 4 & -4 \\ 1 & 7 & -4 & -5 & 7 & -7 & 6 & 2 & -4 & -9 & -3 & 8 \\ 3 & 1 & 7 & -5 & 1 & 0 & -4 & -6 & 5 & -7 & -3 & 3 \\ -3 & 2 & -6 & -1 & -2 & 4 & -8 & 8 & -2 & 0 & 3 & 7 \\ 5 & -3 & -2 & 0 & 2 & -8 & -7 & 9 & -8 & 7 & -7 & -2 \\ -3 & -4 & 9 & -9 & -8 & 9 & -7 & 5 & -1 & -5 & 0 & 6 \\ 4 & 0 & 7 & -6 & -1 & -2 & 5 & 1 & 6 & 6 & 0 & 0 \\ 3 & -9 & 0 & -9 & 6 & 7 & -1 & 9 & -9 & -9 & -9 & 4 \end{bmatrix}$$

The flag for “Large Matrices” (flag -110) is set on the HP49G when calculations include the 12x12 matrix Mat4.

All calculations are done in exact mode and the real matrices consist of integers.

Expression	Result	TI89	HP49G
Mat1 + Mat1	5x5 Matrix	< 1	0,35
Mat1 * Mat1	5x5 Matrix	2,00	3,34
Det(Mat1)	$x^5 - 16 \cdot x^4 + 26 \cdot x^3 + 260 \cdot x^2 + 1241 \cdot x - 6452$	8,50	4,15
Det(Mat3)	$-A^3 + 3 \cdot A - 2$	1,00	0,40
$Mat2^{-1}$	5x5 Matrix	2,50	5,12
$Mat3^{-1}$	3x3 Matrix	2,50	3,87
$Mat3^{-4}$	3x3 Matrix	61,50	14,10
$Mat4^{-1}$	12x12 Matrix	66,00	64,87
Det(Mat4)	3293507711796	15,50	18,40
Mat4 * Mat4	12x12 Matrix	4,50	9,99
EigenVectors(Mat3)	3x3 Matrix	Fail	9,65

Notes:

- The TI89 isn’t able to compute symbolic eigenvectors and eigenvalues (numeric ones only).
- The command EGV on the HP49G computes the matrix of eigenvectors, but also computes the resulting vector of eigenvalues. In all, the HP has a lot of advanced symbolic matrix commands that the TI hasn’t, e.g. HADAMARD, SCHUR, GAUSS, SYLVESTER, JORDAN and PCAR.

NUMERIC MATRIX TESTS

For the numeric matrix tests, Mat2 and Mat4 from the above test are used. The TI89 is in APPROXIMATE mode, and the expressions contain real numeric matrices on both machines;

Expression	Result	TI89	HP49G
Det(Mat2)	6452	1,00	0,43
$Mat2^{-1}$	5x5 Matrix	2,50	1,08
$Mat2^{30}$	5x5 Matrix	13,00	2,10
Det(Mat4)	3293507711796	13,50	2,65
$Mat4^{-1}$	12x12 Matrix	38,00	7,40
Mat4 * Mat4	12x12 Matrix	9,50	2,31
EigenValues(Mat2)	Complex 5d vector	4,50	15,25
FFT(Random 16x16 Matrix)	16x16 Matrix	Fail	11,41
IFFT(Random 16x16 Matrix)	16x16 Matrix	Fail	11,62

Notes:

- The TI89 isn't able to compute Fast Fourier Transforms (or the inverse). This is essential in every aspect of high-speed data sampling on limited hardware. The HP49G seem to be able to do FFT on roughly 25 real-time samples/second – very impressive.

These two matrix tests include all the commonly used matrix operations on various sized matrices, numeric as well as symbolic.
The TI89 is indeed very fast on symbolic matrices, with a couple of weird exceptions though. A shame it doesn't do symbolic eigenvectors. When it comes to numeric matrices, the tables are very much turned – the HP practically whips the TI, again with an oddity; The HP doesn't seem to like numeric eigenvalues.
All in all, this is a tie. If you never use large numeric matrices, it's a win for the TI89 (We would like to say a comfortable win, but there is the very weird situation regarding large negative powers).

INTEGER FACTORIZATION TESTS

Command: TI89 → FACTOR
HP49G → FACTOR

Number	Factorization	TI89	HP49G
38200901201	89 * 11551 * 37159	4,00	2,07
2152302898747	6763 * 10627 * 29947	5,00	3,04
3474749660383	1303 * 16927 * 157543	3,00	1,82
10710604680091	3739 * 18691 * 153259	3,50	4,41
341550071728321	10670053 * 32010157	110,00	43,38
2781632830326137	2781637 * 999998501	41,00	72,45
4498414682539051	46411 * 232051 * 417691	9,00	5,81
6646915915638769	7309 * 321553 * 2828197	15,00	17,14
19041454064475577499	5113 * 3724125574902323	8,00	11,95
46225034404351510471	11969 * 13687 * 98641 * 2860577	17,00	14,88
49444366451936478203	2647 * 398609 * 46861455661	8,00	49,40
148281001870245897403	11779 * 12588590022094057	9,00	16,73
1856275381560031982621	667333 * 2781637 * 999998501	65,00	81,51
16164153678397110532509	3691 * 43793426384169901199	12,00	17,84
130529377836972488251268578591	2647 * 3691 * 5113 * 11779 * 398609 * 556517681	19,00	40,56
(2^127-1): 170141183460469231731687303715884105727	170141183460469231731687303715884105727	50,00	54,73
(2^128-1): 340282366920938463463374607431768211455	3 * 5 * 17 * 257 * 641 * 65537 * 274177 * 6700417 * 67280421310721	220,00	146,66

Notes:

- Both machines ran into > 1000 second expressions during testing. These are not shown here, since it's obvious that when a number (or its factors) reaches a given size, the relatively slow processors that these machines have will not suffice.

During the testing it became clear that the stronger CPU of the TI89 was significant. It seems that when prime numbers reach a certain size, the most powerful CPU wins. At the same time luck (or a cunning algorithm) can haul home an odd win. Considering the Saturn is only about one fourth as strong as the Motorola, it runs at a steady pace, winning a surprising 40% of the integer factorizations – it seems that as the integers get smaller, the more certain the Saturn is to finish first!
What could be a key factor though, is the fact that the HP is loaded with a beta ROM. The fact that the latest version of Erable that runs on the HP48 series is about 2-5 times faster than the HP49G at this, implies that we have big improvements in this area yet to be seen. Mika Heiskanen and Bernard Parisse of HP explains that Erable for the HP48 uses HEX integers in integer factorizations, and that the HP49G uses BCD encoded integers to comply with the rest of the CAS – therein lies the speed penalty. This might be changed in a future ROM release to improve speed, but it's not a major concern at the moment. As for now; a clear win for the TI89.

SOLVE TESTS

Command: TI89 → SOLVE(,{ vars })
HP49G → SOLVE [vars]

Equations	Solutions	TI89	HP49G
$\left(\frac{x\cdot 2}{A\cdot\frac{B}{2}}\right)^2\cdot\frac{\ln(A)}{2}=\frac{1}{(B\cdot 4)^2}$	$x=\frac{\sqrt{2\cdot\ln(A)}\cdot A}{16\cdot\ln(A)}$ or $x=-\frac{\sqrt{2\cdot\ln(A)}\cdot A}{16\cdot\ln(A)}$	1,00	20,27
$2\cdot\ln(x)^2-3\cdot\ln(x)=5$	$x=\exp(-1)$ or $x=\exp\left(\frac{5}{2}\right)$	1,00	3,30
$x^5-2{,}2\cdot x^3+9=0$	$x=-1.88339169416$	7,50	3,02
$((x-1)\cdot(x-2)\cdot(x-3)\cdot(x-4)\cdot(x-5))^2=0$	$x=1$ or $x=2$ or $x=3$ or $x=4$ or $x=5$	6,00	3,80
$17\cdot x^{10}+2\cdot x^9+9\cdot x^8+x^7+10\cdot x^6+9\cdot x^5+14\cdot x^4$ $+17\cdot x^3+12\cdot x^2+6\cdot x+1=0$	$x=-0.263804061472$ or $x=-0.631621261851$	50,00	8,65
$\sin\left(x-\frac{\pi}{2}\right)=\pi$	$x=\mathrm{acos}(\pi)-(2\cdot n-1)\cdot\pi$ or $x=-\mathrm{acos}(\pi)+(2\cdot n-1)\cdot\pi$	Fail	5,12
$x^2+y^2-3=7$ and $x-2\cdot y=6$	$x=\frac{6+2\cdot\sqrt{14}}{5}$ and $y=\frac{-12+\sqrt{14}}{5}$ or $x=\frac{6-2\cdot\sqrt{14}}{5}$ and $y=\frac{-12-\sqrt{14}}{5}$	3,00	2,47
$t-z+y=53$ and $y+x-6=22$ and $x\cdot 6-z=46+t$ and $t-x+y=3$	$t=-\frac{133}{3}$ and $z=-\frac{179}{3}$ and $x=-\frac{29}{3}$ and $y=\frac{113}{3}$	2,00	3,33

Notes:

- In the above examples, only real solutions are solved for, and multiple similar solutions are not listed. This is done to reduce the size of the table (due to many solutions of large order polynomials).
- It seems that for most equations, solving for all solutions (real and complex) on both machines takes not much longer (if at all) than solving for real solutions only.

- On the HP49G, you get prompted, while solving, for if you want to change into approx. mode (if there's numeric solutions that can't be expressed purely symbolic) or into complex mode (if there's complex solutions). If you answer no to any of these prompts, you'll only be presented with the chosen solutions.

The TI89 has a very capable solver. On many points it's far superior to and much faster than the HP equivalent. The tests above reveal some of these points, but again, we're sad to say, some implementation quirks from the Derive engine yields some odd results once in a while. A good feature which is greatly missed on the HP is the ability to solve for numerous numeric solutions in one go. On the '49 you need to supply guesses to the ROOT function, or plot the equation, if the SOLVE command fails.
It's also these capabilities of the TI89 that are responsible for most of the confusion – the Derive engine is so keen on getting all the solutions, that occasionally it throws up garbage such as some periodic functions that gets solved sporadically (not yielding some roots and the period, but a handful of randomly (or at least erratic) chosen solutions along the function).
On the topic of polynomials, there's really no match for the HP – it's unbeatable. Not only in regard to the examples supplied here, but in a general sense. It has a whole range of advanced tools to pick from – ranging from CHINREM (solves a system of simultaneous polynomial congruences in the ring Z[**x**]) over HERMITE, TCHEBYCHEFF and LEGENDRE, to HORNER, LAGRANGE and a whole variety of division and modulo commands.
Systems of linear equations are solved both fast and efficiently by both machines, but the HP seems to have an edge on non-linear equations as long as they are polynomials (the internal Gröbner-bases engine is rumored to be available to the user in a later stage).
Considering that both gets the job done most of the time, and appreciating that the HP49G can be in just as much trouble as the TI89 seems to be sometimes, it's a tie. Should the result be split up in solving non-polynomial equations and polynomials, the TI89 would have a clear win in the first case and the HP49G a clear win in the second (the TI89 can solve some non-polynomic multivariate simultaneous non-linear equation, where they have to be polynomic for the HP49G).

Conclusion

The King is dead, long live the King! That could be our final words, since the TI89's deserved place on the throne is now definitively over.
The TI89 still has it advantages, but they are quickly diminishing in the light of the new kid on the block – don't kid yourself, it's absolutely a fine calculator, it's just not the best anymore. When that's said, we must add that the new HP implies a whole new way of thinking. It's a tool and need to be used as such – you need to know what your goal is before you can select the proper command or application. On the other hand there is a lot of headroom for experimenting. This is a win/win situation for students, that are a new target group for HP; they will be able to learn to use it quickly by utilizing among other things the choice of algebraic entry (though they might want to switch over to RPN pretty fast) and the new step-by-step routines. The demand for knowing your task before being able to solve it will help students in learning new stuff, while at the same time keeping a constant flow of new discoveries – the tool will grow as they do. In our opinion something not seen since the release of the HP48G-series back in 1993.
If the '49 should have an Achilles' heel, it's got to be the beta state of the ROM. It's very stable and we didn't experience severe problems with it, but some have reported different bugs and quirks still present in it. We're confident, though, that HP will solve problems within the ROM quickly and efficiently as they always have. At the same time there will continue to be added new functionalities to the machine, something TI has failed graciously in over the past years, even when they had the opportunity with flashROM long before HP.
This is just a short test of the CAS of the two machines, it's by no means a complete run-through of all functionalities (just consider the full-blown text editor with editable fonts, italic, bold, inverted and underlined text, search & replace, cut, copy & paste….the comprehensive statistics and finance utilities….the high-speed I/O capabilities by means of a standardized RS232 port (though it lacks the IR of the HP48)….the EquationWriter, MatrixWriter, units, number bases, time management and the 3D wireframe plotter that rotates 8 frames/second….the built-in support for four programming languages, including compilers and debuggers for assembly, SysRPL, HPBasic and UserRPL…the customizable display, keyboard, menus and so on…..) – that would be to big a task for us to do right now, but take our word for it;

The HP49G is worth a big consideration if looking for a new tool, toy or studying aid.
Now we can’t wait for TI to make their next move – they’ve got the people and the hardware, let’s see them use it.

Update as of 6th January 2000:

It is now proven that the Achilles’ heel of the HP49G is it’s documentation – or lack thereof. While the machine is spot-on for the people who are used to the HP line of calculators, it’s hard to get used to for newcomers due to it’s severe lack of documentation – this has to improve before the machine will be a hit. The ROM situation has very much improved as expected. The current release version is 1.16, and the current beta is 1.17.4 – both stable and with even more functions than previous versions.
In comparison the AMS 2.xx for the TI89 has been very much a disappointment. Though fixing some bugs it’s not the major leap forward in functionality as expected – a shame. We see forward to the real improvements in a not yet released ROM. Come on TI; There’s a Flash ROM in the machine – use it.