

[2.10] CAS square root simplification limitations

The 89/92+ CAS (computer algebra system) rarely treats the square root operator as identical to raising the argument to the 1/2 power, because, in general, the two operations are not equivalent. For example,

$$x^{\frac{3}{2}} - \sqrt{x^3}$$

does not simplify to zero. However, the expression does simplify to zero if we restrict x, in other words, these expressions

$$x^{\frac{3}{2}} - \sqrt{x^3} \mid x > 0 \quad \text{or} \quad x^{\frac{3}{2}} - \sqrt{x^3} \mid x \geq 0$$

both correctly return zero.

If you write programs that manipulate symbolic expressions, you need to consider the domain of the variables. For example, for $x < 0$,

$$\left(x^{\frac{1}{2}}\right)^3 = -(-x)^{\frac{3}{2}} \cdot i \quad \text{and} \quad (x^3)^{\frac{1}{2}} = \sqrt{-x^3} \cdot i$$

If $x = -1$, then the first expression gives $-i$, and the second is i . Both expressions *do not* simplify to $x^{3/2}$, because the identity $(x^a)^b = x^{a*b}$ is not true for non-integer exponents a and b .

You also need to consider the CAS' behavior for x^n when $x = 0$. For example:

$$\begin{aligned} 0^n \mid n > 0 &= 0 \\ 0^n \mid n = 0 &= 1, \text{ with the warning } 0^0 \text{ replaced by } 1 \\ 0^n \mid n < 0 &= 0^{\text{undef}} \end{aligned}$$

(Credit to Carlos Becker for straightening me out on this)