[2.24] Infinity^0 evaluates to 1

The TI-89/TI-92+ CAS evaluate infinity to the zero power as one. In general, x^0 is 1, for x not equal to zero, however, since infinity is not a number, the usual rules of arithmetic do not necessarily apply. Different CAS systems may return different results, for example, the HP-49G considers the result undefined or 1, depending on flag settings, and the HP-48, Mathematica and Maple all return 1.

One plausible rational for setting $\infty^0 = 1$ is that

$$\lim_{x \to 0} \left(\frac{1}{x}\right)^x = 1$$

however, 1/x is not infinity, it is undefined.

There are actually different 'degrees' of infinity. A denumerably infinite set is equivalent to the set of all natural numbers. The German mathematician Georg Cantor defined an infinite set as one which can be put in one-to-one correspondence with a proper subset of itself. Cantor denoted the 'size' of a denumerably infinite set as $\aleph_0$ which is called aleph-null, aleph-zero, or aleph-naught. aleph is the first letter of the Hebrew alphabet. There are larger infinite sets, which are not in one-to-one correspondence with the set of natural numbers. These are designated $\aleph_1, \aleph_2, \aleph_3 ...$ and so on. Aleph-zero, aleph-one and so on, are called the cardinal numbers of infinite sets, where each set has a higher degree of infinity. These cardinal numbers are called transfinite numbers. If you are interested in learning more about transfinite numbers, I recommend chapter 7 of *Mathematics from the Birth of Numbers*, by Jan Gullberg, as an easy introduction. Unfortunately, Cantor's ideas were so radical that he encountered intense criticism from his peers, suffered from nervous breakdowns, and died in a mental institution. Cantor proved these results:

$$\aleph_0 + \aleph_0 = \aleph_0 \qquad (\aleph_0)^2 = \aleph_0 \qquad 2^{\aleph_0} = \aleph_0^{\aleph_0} > \aleph_0$$

In any event, the TI-89/TI-92+ give this warning:

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Warning: ∞^Ø or undef^Ø replaced by 1
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I emailed TI Cares about this operation. Here is the response:

"An infinity symbol is necessary for doing limits, and if it is allowed as an argument or result of the limit function, then something must be done when it is combined with other expressions.

"What the TI-92 does in such compositions is consistent with extended analysis. It is also consistent with the ANSI standards for IEEE floating point arithmetic, and is the same as other CAS systems that treat infinity. For example,

inf - inf => undef

1/inf => 0

abs(1/0) => abs(+-inf) => inf

and

inf - 100 => inf

"For transformations such as the latter, it might help to think of inf as representing a whole set of numbers rather than a single one.

"In order to obtain sharp results in computer algebra, it is important to discard as little information as possible throughout each computation. For example, this is absolutely crucial for internal computations of limits, which is done recursively via rules such as the limit of an absolute value is the absolute value of the limit of the argument.

"It is true that at the more elementary levels of math education we lump more into the phrase "undefined", which therefore doesn't have a single definition. Rather, it means "I don't want to talk about that yet."

"However, for consistency, the computer algebra must implement one place along this spectrum of sophistication. Unless the place is "discard as little information as is practical", people will be disappointed that the product can't do certain limits, definite integrals, solutions of equations, etc. that are in standard math tables and relevant textbooks.

"Actually, we did weaken the product somewhat to appease the more elementary end of the spectrum: +-inf is necessarily carried internally, but it is degraded to undef during output. This is why abs(1/0) simplifies to inf but 1/0 is displayed as undef: Compositions can be more powerful than stepwise computations such as 1/0 STO foo: abs(foo). This is additional evidence that the only easily explained places on the spectrum are: a) No infinities: only undef as on typical purely numeric calculators; or b) Make it as powerful as is practical. The first choice is simply not an alternative if there is a desire to do symbolic limits, improper integrals, etc.

"I am sorry that a single product can't exactly match the needs of education from beginning algebra through second-year calculus. We chose to make the product as powerful as we could within the constraints on ROM and programming time. Many teachers have found that the excess power for some classes stimulates the students curiosity and provides opportunities for lively discussion. It also allows the students to buy a single calculator that suffices for a succession of courses."

To which I would add this:

A graduate student of Trinity
Computed the square of infinity.
But it gave him the fidgets
To put down the digits,
So he dropped math and took up divinity.

- Anonymous