

[6.49] Taylor() polynomial function finds tangent line

The line tangent to a function $y = f(x)$ at a point $x = a$ is defined by the criteria that the line passes through point $(a, f(a))$ and the slope of the line is the derivative of $f(x)$ at $x = a$. If the tangent line is

$$y = m \cdot x + n \quad [1]$$

then $m = \frac{d}{dx} f(x) \mid x = a$ [2]

and if $f(a) = b$, we find n with $n = b - m \cdot a$ [3]

The tangent line is the first-order Taylor series polynomial at $x = a$, so we can use the built-in *taylor()* function to find the line equation. The syntax for *taylor()* is

```
taylor(expr, var, order [,point])
```

where *expr* is the expression, *var* is the independent variable, *order* is the polynomial order, and the optional *point* argument is the point at which to find the series expansion. To find the tangent line, we set *order* = 1 and *point* = a . For example, if

$$y = 2 \cdot x^2 - 7 \cdot x + 1$$

and we want to find the tangent line at $x = 3$, then use

```
taylor(2*x^2-7*x+1,x,1,3)
```

which returns $5(x - 3) - 2$. Use *expand()* on this result to put it in the more common form of $5x - 17$.

We can also use this method to find symbolic results. For example, suppose

$$y = \ln(x^2)$$

and we want to find the tangent line at $x = a$, then use

```
expand(taylor(ln(x^2),x,1,a))
```

which returns $\frac{2 \cdot x}{a} + \ln(a^2) - 2$

This method fails if the CAS cannot find the Taylor polynomial for the expression. This is the case for complicated user-defined functions. In this case, you can find the derivative numerically (see, for example, tip 6.26), then solve for the tangent line constant with equation [3] above.